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BLOCKING ANALYSIS FOR TIME-SPACE SWITCHED ALL-OPTICAL NETWORKS

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ABSTRACT

Optical time division multiplexing (OTDM) allows multiple traffic streams to share the bandwidth of a wavelength efficiently. In this paper, we present a new analytical model, based on the inclusion-exclusion principle from combinatorics, for evaluating the blocking performance of time-space switched optical networks with fixed routing and random wavelength/timeslot assignment. This model can be used to analyze networks with arbitrary topologies and traffic patterns. The accuracy of the proposed analytical model is validated through simulations.

KEY WORDS

Blocking probability, WDM/TDM switching, optical networking

1 Introduction

In wavelength-division multiplexing (WDM) networks, the optical spectrum is divided into many different channels, and each channel corresponds to a different wavelength which can operate at the peak electronic speed. However the bandwidth of a single wavelength is too large for some traffic. Time division multiplexing (TDM) techniques have been developed to improve the elasticity and granularity of a path in WDM networks by dividing a wavelength into several time slots and multiplexing traffic on the wavelength. In TDM networks, time slot interchangers (TSI), which can be accomplished in the electronic domain by shift registers, are widely used to rearrange the order of the time slot of traffic passing through them.

Optical TDM (OTDM) has been studied for years now, at the component and system level. Recent advances of optical switching technology have shown the possibility of realizing fast all-optical switches, which can be reconfigured in less than a nanosecond [1] [2]. The use of such fast switches along with fiber delay lines as optical TSIs has opened up the possibility to realize optical time switched networks. The bandwidth granularity offered by an OTDM network is determined by the duration of a time slot, which, in turn, depends on the speeds at which the switching can be accomplished. In WDM networks, the data remain in the optical domain throughout their path. Such paths are termed lightpaths. In OTDM networks, the paths are named optical trails to distinguish them from the lightpaths in WDM networks.

OTDM networks can be classified into two categories: dedicated-wavelength OTDM networks (DW-OTDM) and shared-wavelength TDM (SW-OTDM) networks. In DW-OTDM networks, dedicated optical trails are established between each source-destination pair. The optical trails occupy the same time-slots over the same wavelength along their path. No switching is performed in the time domain between the source node and the destination node. On the other hand, in SW-OTDM networks, an optical trails can be established over the same wavelength but using different time-slots along its route. Benefited from time-slot switching, SW-OTDM can achieve higher performance in terms of reducing blocking probabilities than DW-OTDM. However, the use of fiber delay lines in SW-OTDM networks introduces additional propagation delay and increases the cost and complexity of OTDM switches.

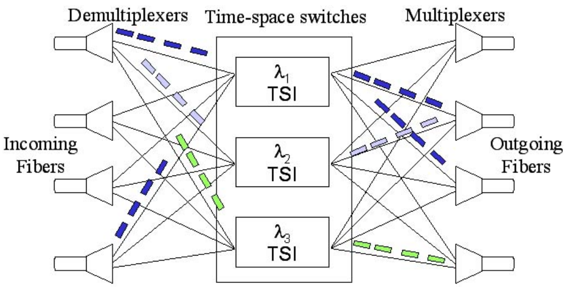

Figure 1: Node architecture in SW-OTDM/WDM networks

Routing individual slots dynamically requires processing header information hop-by-hop. This can be done in electrical domain, like Optical Burst Switching (OBS), but not in optical domain because of the immatureness of optical processing and storage techniques. Therefore, OTDM switched networks are expected to be circuit-switched in nature. Like in other circuit-switched networks, blocking probability is a primary performance evaluation metric of OTDM switched networks. A distinctive character of OTDM switched networks is the *wavelength continuity constraint* in routing of optical trails. In OTDM networks without wavelength conversion, a multi-hop traffic flow can only be switched from one time-slot to another within the same wavelength, as shown in Fig. 1.

The problem of computing blocking probabilities for WDM networks under static routing with random wavelength allocation and with or without wavelength converters has been studied extensively in [3] [4] [5]. However, there is not much analytical work for OTDM networks. DW-OTDM networks are logically equivalent to single fiber WDM networks. Thus the previous analytical models for WDM networks can be used for analyzing DW-OTDM without any changes. It has been shown that SW-OTDM networks are logically equivalent to WDM networks with multiple fibers on each link and no TDM [7]. Unfortunately, the analytical models for multi-fiber WDM networks in the literature, including a recently published one [3], are complicated and not accurate. In [4], blocking performance has been analyzed for one isolated multi-hop path in single-fiber multi-wavelength TDM-switched networks. However, this model did not consider the contention of bandwidth from other traffic requests. Therefore, it cannot capture the dynamic nature of network traffic and cannot accurately calculate the network-wide blocking probabilities.

Future wide area networks are most likely to be all-optical OTDM/WDM networks with tens or hundreds of nodes and tens of wavelengths/timeslots multiplexed on each link. Because of high computational complexity, the models in the current literature could not apply satisfactorily to the analysis of such large networks. Thus the objective of this work is two-fold. One is to develop a technique applicable to arbitrary topologies which is computationally tractable. The other is to give reasonable estimates of blocking probabilities for design purposes and the analytical study of issues like benefits of TSI, wavelength conversion and so on.

In this paper, a new analysis model is proposed for performance evaluation of SW-OTDM/WDM networks. The DW-OTDM switched networks can be viewed as a special SW-OTDM switched networks, which only have one time-slot per wavelength. Particularly, we investigate the dimensioning issues of SW-OTDM switched networks though studying the effect of TSI on network performance. The remainder of the paper is organized as follows: Section II presents the analytical model of SW-OTDM networks. Section III discusses the performance results obtained using the analytical model. Section IV concludes the paper.

2 Analytical Model

Typically, in a network, the blocking probabilities of the paths and arrival rates to a link are coupled to each other by the fact that the blocking determines the traffic carried by the network and the carried traffic in turn determines the blocking. Our analytical model consists of three parts: a traffic model, an OTDM model, and a lightpath model. The Traffic model assumes that the idle time-slot distribution on a link can be described by the state-dependent routing model first proposed by Kelly and developed in [6]. The same model is also used in [5]. The traffic model consists of a set of equations that determine the traffic offered to each link at time-slot level according to the path blocking probabilities. On the other hand, due to the wavelength continuity constraint, for multi-hop connection requests, an optical trail has to use the same wavelength along its route. A multi-hop connection could be blocked even if there are idle time-slots on each link of the route but in different wavelengths. Thus, the lightpath model consists of a set of equations that determine the path-blocking probabilities according to the offered traffic on each link at the wavelength-level. The OTDM model converts traffic load from the time-slot level to the wavelength level, and thus establishes a connection between the traffic model and the lightpath model. This leads to a set of coupled non-linear equations which must be solved to obtain the blocking probabilities. Iterative algorithms are designed to obtain the approximate solutions in most analysis methods, including ours.

2.1 System Parameters and Assumptions

1. The network consists of N nodes connected by J links in an arbitrary fashion.
2. Each link has the same B wavelengths, each wavelength consists of K time-slots, thus each link has a fixed capacity of C time-slots, $C = K \times B$.
3. Calls for a node pair S arrive according to an independent stationary Poisson process with rate λ_s. Each call requires a full time-slot on each link of its path.
4. The duration of each call is exponentially distributed with a mean of one unit ($1/\mu = 1$).
5. Traffic on one time-slot can only be switched onto another time-slot in the same wavelength.
6. The wavelength and time-slot assigned to a route is chosen uniformly randomly from the set of idle wavelengths and time-slot. The assumption makes all wavelengths and time-slots identical and the analysis tractable.
7. We assume fixed routing. This means that each node-pair has exactly one pre-determined route.
8. The state of a wavelength on link j is independent of the state of wavelengths on link $j-1$. In other words, the sets of idle/occupied wavelengths on links are independent. This is also called link-load independence assumption.

2.2 Traffic Model

Definitions:

X_R The random variable representing the number of idle time-slots on route R.

X_j The random variable representing the number of idle time-slots on link j.

$q_j(m)$ The probability that exactly m time-slots are idle on link j.

$\alpha_j(m)$ Given exactly m idle time-slots on link j, the time until the next call setup on j is exponentially distributed with parameter $\alpha_j(m)$.

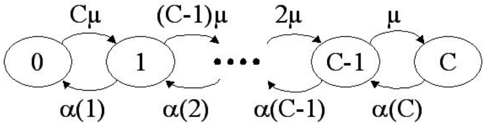

Figure 2: Markov chain for the distribution of idle wavelengths on a fiber

$$q_j(m) = \Pr\{X_j = m\} \tag{1}$$

The number of idle time-slots on link j can be viewed as a birth-death process. The arriving and serving behavior on link j forms an M/M/C/C system and the corresponding Markov chain is illustrated in Fig. 2. Since all the states in the Markov chain are ergodic and hence, the equilibrium state distribution of the chain can be derived as follows:

$$q_j(m) = \frac{C(C-1)...(C-m+1)}{\alpha_j(1)\alpha_j(2)...\alpha_j(m)} q_j(0), \tag{2}$$

where

$$q_j(0) = [1 + \sum_{m=1}^{C} \frac{C(C-1)...(C-m+1)}{\alpha_j(1)\alpha_j(2)...\alpha_j(m)}]^{-1}. \tag{3}$$

$\alpha_j(m)$ is obtained by combining the contributions from the request streams to routes which traverse j, as follows:

$$\alpha_j(m) = \sum_{R:j\in R} \lambda_R \Pr\{X_R > 0 | X_j = m\} \, for \; m = 1, ..., C. \tag{4}$$

2.3 OTDM Model

Definitions:

β_j The number of idle time-slots on link j.

$\beta_j(b)$ The probability that exactly b wavelengths are idle on link j. A wavelength is idle if any of its time-slots is idle.

The conditional probability that b wavelengths are idle in link j under the condition that N time-slots are idle in link j can be obtained by using inclusive-and-exclusive theory in combinations and permutations.

$$Pr\{\beta_j = b | X_j = N\} = 0 \;\; if (N < b \; or \; N > bK)$$

$$= \frac{\binom{B}{b} \times \binom{bK}{N}}{\binom{BK}{N}} \;\; if((b-1)K < N \le bK)$$

$$= \frac{\binom{B}{b}\binom{bK}{N}}{\binom{BK}{N}}$$

$$+ \frac{\sum_{z=max\{1,K/N\}}^{b-1} (-1)^{b-z} \binom{B-z}{b-z}\binom{B}{z}\binom{zK}{N}}{\binom{BK}{N}}$$

$$if (b \le N \le (b-1)K)$$

$$\beta_j(b) = \sum_{N=1}^{C} Pr\{\beta_j = b | X_j = N\} Pr\{X_j = N\} \tag{5}$$

2.4 The Lightpath Model

Definitions:

B_R Blocking rate of calls on route R

$Y_{i,j}$ The random variable denoting the state of wavelength i on link j. $Y_{i,j} = 0$ if wavelength i is idle on link j; $Y_{i,j} = 1$, otherwise.

$\gamma_{i,j}$ The probability that a fixed set of i wavelengths is idle on link j.

g_i^R The probability that a fixed set of i wavelengths is idle on route R.

For 1-hop routes, the blocking probabilities can be obtained from:

$$B_R = q_j(0) = \beta_j(0) \tag{6}$$

The blocking probability of a multi-hop route R is the probability that there is no wavelength (or time-slot) which is idle on all the links used by R. We have

$$B_R = Pr\{X_R = 0\} = 1 - Pr\{X_R > 0\} \tag{7}$$

Using inclusion-exclusion principle and the assumption of random wavelength assignment, we have

$$Pr\{X_R > 0\} = \sum_{i=1}^{B} (-1)^{i-1} \binom{B}{i} g_i^R \tag{8}$$

Under the assumption of link-load-independency, the probability that a fixed set of i wavelengths is idle on a multi-hop route R is

$$g_i^R = \prod_{j:j\in R} \gamma_{i,j}, \tag{9}$$

where

$$\gamma_{i,j} = Pr\{Y_{1,j} = 0, Y_{2,j} = 0, ..., Y_{i,j} = 0\}. \quad (10)$$

From the assumption of link-load independency, we have the following conditional probability of a fixed set of i wavelengths is idle on link j provided that there are a total of b idle wavelengths on link j.

$$Pr\{Y_{1,j} = 0, ..., Y_{i,j} = 0 | \beta_j = b\} = \frac{\left(\begin{array}{c} b \\ i \end{array}\right)}{\left(\begin{array}{c} B \\ i \end{array}\right)} \quad (11)$$

According to the law of total probability, we have

$$\gamma_{i,j} = \sum_{b=1}^{B} \beta_j(b) \frac{\left(\begin{array}{c} b \\ i \end{array}\right)}{\left(\begin{array}{c} B \\ i \end{array}\right)} \quad (12)$$

Therefore, the state dependent blocking probability of route R can be obtained by

$$Pr\{X_R > 0 | \beta_j = b\} = \sum_{i=1}^{B} (-1)^{i-1} \left(\begin{array}{c} B \\ i \end{array}\right) g_i^R(X_j = b), \quad (13)$$

where

$$g_i^R(\beta_j = b) = \left(\prod_{j:j\in R} \gamma_{i,j} \right) \frac{\left(\begin{array}{c} b \\ i \end{array}\right)}{\left(\begin{array}{c} B \\ i \end{array}\right)}. \quad (14)$$

Given m idle time-slots on link j and the lightpath model, the equation (4) can also be defined as follows :

$$\alpha_j(m) = \sum_{R:j\in R} \sum_{b=1}^{B} Pr(X_R > 0 | \beta_j = b) Pr\{\beta_j = b | X_j = m\}. \quad (15)$$

2.5 Analysis of a Network

Based on the lightpath model, the network-wide blocking probability can be obtained by the ratio of the total blocked load versus the total offered load, i.e.,

$$P = \frac{\sum_{s=1}^{N(N-1)} \lambda_s Pr\{X_s = 0\}}{\sum_{s=1}^{N(N-1)} \lambda_s}. \quad (16)$$

2.6 Computation of blocking probability

From the above analysis, a set of non-linear coupled equations has been obtained for the computation of blocking probabilities. An iterative algorithm can be developed accordingly to find the solution by repeated substitution. In practice, the solutions converge in a few iterations for a variety of topologies. The method of iterative substitution is described as follows:

1. For all routes R, initialize $\tilde{B}_R$ to zero. For $j = 1, ..., J$, initialize $\alpha_j(0) = 0$ and $\alpha_j(m) = \sum_{R:j\in R} \lambda_R, m = 1, ..., C$.
2. Determine the idle capacity distribution of all links $q_j(\cdot), j = 1, ..., J$, using equations (2) and (3).
3. Calculate $\gamma_{i,j}$ for all links, $j = 1, ..., J$ and $i = 1, ..., B$ using equation (12).
4. Update $\alpha_j(\cdot)$ using equations (5) (8) and (15).
5. Calculate B_R for all routes. If $\max_R |B_R - \tilde{B}_R| < \epsilon$ then terminate. Otherwise, let $\tilde{B}_R = B_R$ and go to 2).

2.7 Computational Complexity

One of the main objective of this paper was to propose analytical models with reduced complexity to enable the study of large networks. The computational requirement of the proposed analytical model is $O(N^2JB^2C)$. The computational complexity of the technique presented in [3] is $O(N^2JCB^{C/B+2})$ for fixed routing, which limits its applicability to small networks. Although the proposed OTDM model is less complex than the multi-fiber model in [3], the OTDM model can achieve higher accuracy than the multi-fiber model, because in multi-fiber model the carried link-loads are approximated by offered link-loads, which would introduce large errors when traffic load is high.

3 Numerical Results

In this section we demonstrate the accuracy of our analytical techniques by comparing analytical results to simulation results. Simulation results are plotted along with 95% confidence intervals estimated by the method of replications. The number of replications is 30, with each simulation run lasting until each type of call has at least 100,000 arrivals. For the analytical results, the iterative algorithm terminates when all blocking probability values have converged within 10^{-5}.

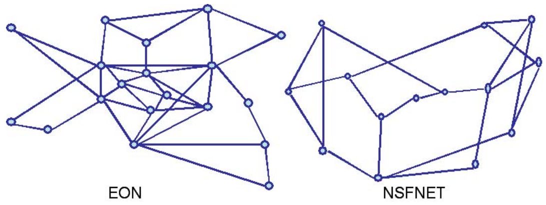

Figure 3: Network topologies

Simulation experiments are conducted on the 14-node NSFNET and 19-node EON network topologies, which are shown in Fig.3. In the simulations, the call requests arrive to the network following a Poisson process, and the call-holding time is exponentially distributed. We assume that all the source-destination node pairs have the same traffic load in Erlang. Each fiber link has fixed capacity (32 time-slots channels). Simulations are conducted with different wavelength-timeslot combinations of the same link capacity: 2-wavelength-16-timeslot, 4-wavelength-8-timeslot, 8-wavelength-4-timeslot, and 16-wavelength-2-timeslot. We define the *time-slot-wavelength ratio* (TSWR) as the ratio of number of time-slots per wavelength over total number of wavelengths per link. Fixed shortest path routing is used to calculate the shortest path (in hop-counts) for each node pair. The granularity of a call connection is a optical-trail, which occupies one time-slot on each link along the route.

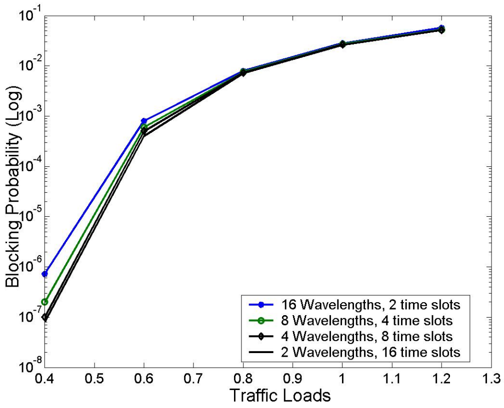

Figure 4: Analysis results of network-wide blocking probability versus network load for EON network

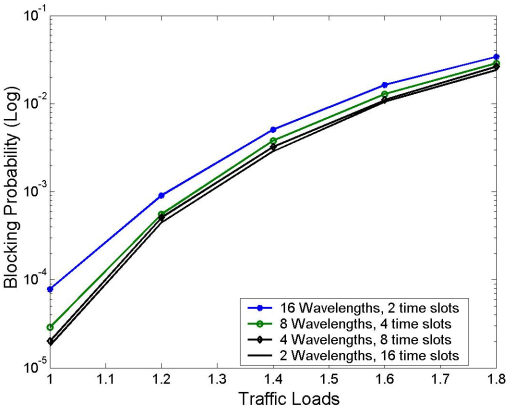

Figure 5: Analysis results of network-wide blocking probability versus network load for NSFNET network

Fig. 4 and Fig. 5 demonstrate the numerical results obtained from the proposed analytical models for the EON network and the NFSNET network, respectively. It can be observed that the network-wide blocking probabilities increase as the network load becomes heavier. Given the same link capacity, it can also be observed that it decreases as TSWR increases. However, the performance gains resulting from the increase of TSWR cannot be improved further once the number of time-slots per wavelength increases above a certain value for both networks.

The switches with higher TSWR are required to operate at higher space switching speed. For example, the switching speed of a 2-wavelength-16-timeslot switch has to be 4 times higher than that of an 8-wavelength-4-timeslot switch in order to achieve the same link capacity. On the other hand, the switches with lower TSWR require larger number of transmitters and receivers although each of them can work at lower speed. For example, a 2-wavelength-16-timeslot switch needs only one fourth the transmitters and receivers compared to an 8-wavelength-4-timeslot switch that has the same capacity.

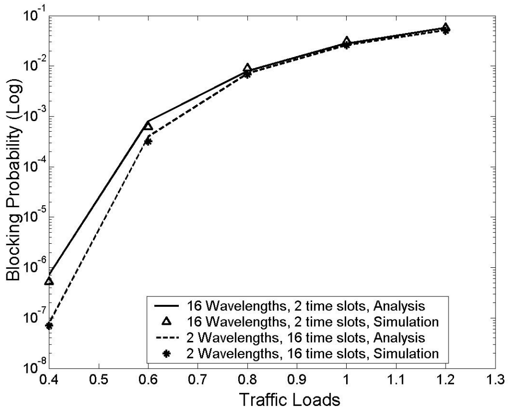

Figure 6: Comparison of numerical results for EON network

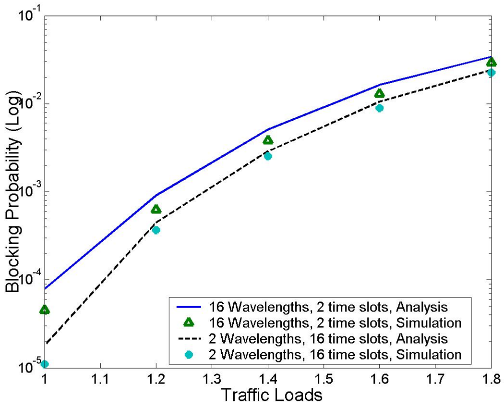

Figure 7: Comparison of numerical results for NSF network

Fig. 6 and Fig. 7 compare the numerical results obtained from the analytical models to those from simulation experiments for the EON and the NSFNET networks, respectively. The numerical results of EON conform closely to the simulation results. However, for the NSFNET topology, there are noticeable difference between analytical results and simulation results. This can be explained by the assumption of link-load independency in this analytical model. For networks with low node-degree and low TSWR, the link-load independency model cannot accurately capture the wavelength usage in the network. As indicated in [5], the link-load independency assumption can be adapted to a link-load correlation assumption, thus a more accurate estimation of the blocking probabilities can be obtained but with higher computational complexity.

4 Conclusion

An analytical model has been developed for evaluating the blocking performance of all-optical time-space switch WDM networks. The analytical model can also be used to study performance of multi-fiber WDM networks with/without limited-range/full wavelength conversion because of the logical equivalence of these networks. Using the analytical model, the network-wide blocking probability is derived from the traffic model, the OTDM model and the lightpath model. Compared to previous analytical models in the literature, our model has relatively low complexity. The comparison between numerical and simulation results indicates that the computational model is accurate in calculating the blocking performance of all-optical OTDM/WDM networks. The numerical results have also shown that significant performance gain can be achieved in WDM networks using OTDM.

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